

**Mathematics Applications  
YEAR 12**

**Investigation 3 – Consecutive Numbers**

**Semester 2 2016**

**Time allowed:** 60 minutes

**Marks Available:** 52 marks

**Materials required:** Writing implements, correction fluid/tape or eraser, ruler, Scientific or CAS calculator

**Instructions:**

1. Write your answers in the spaces provided in this Question/Answer Booklet.
2. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

1. (14 marks)  
   The number 6 can be written as the **sum** of 3 **consecutive** whole numbers:

6 = 1 + 2 + 3

Consecutive numbers follow one after another. They are next to each other in order.

Show **all** the possible ways you can write the numbers 2 to 16 as sums of consecutive whole numbers? Note: This will not be possible for all numbers (write ‘does not work’ if so).

|  |  |
| --- | --- |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 | 4+5 and 2+3+4 |
| 10 |  |
| 11 |  |
| 12 |  |
| 13 |  |
| 14 |  |
| 15 |  |
| 16 |  |

1. (4 marks)  
   Which of these numbers could **not** be written as the sum of consecutive whole numbers?

What sequence do these numbers represent?

1. (6 marks)  
   Pick any **6 pairs** of consecutive whole numbers and make a list of their sums: [3]

|  |  |  |
| --- | --- | --- |
|  | Consecutive Numbers | Sum of consecutive numbers |
| Pair 1 |  |  |
| Pair 2 |  |  |
| Pair 3 |  |  |
| Pair 4 |  |  |
| Pair 5 |  |  |
| Pair 6 |  |  |

1. What do the **sums** have in common? [1]
2. Using n and n+1 as your pair, prove your discovery from a), given that 2n+1 must be an odd number. [2]
3. (8 marks)  
   Pick any 6 sets of **3** consecutive whole numbers and make a list of their sums: [3]

|  |  |  |
| --- | --- | --- |
|  | Consecutive Numbers | Sum of consecutive numbers |
| Set 1 |  |  |
| Set 2 |  |  |
| Set 3 |  |  |
| Set 4 |  |  |
| Set 5 |  |  |
| Set 6 |  |  |

1. What do the sums have in common? [1]
2. What is the connection between the **middle number** and the **sum**? [1]
3. How could you calculate 107+108+109 without adding the numbers? [1]
4. If your 3 numbers are n, n+1 and n+2 then prove your discovery in a) [2]
5. (8 marks)  
   Pick any 6 sets of **4** consecutive numbers and list their sums and half their sums: [6]

|  |  |  |  |
| --- | --- | --- | --- |
|  | Consecutive Numbers | Sum of consecutive numbers | Sum / 2 |
| Example | 4 + 5 + 6 + 7 | 22 | 11 |
| Set 1 |  |  |  |
| Set 2 |  |  |  |
| Set 3 |  |  |  |
| Set 4 |  |  |  |
| Set 5 |  |  |  |
| Set 6 |  |  |  |

1. If you write the numbers as (*n* −1) + *n* + (*n* +1) + (*n* + 2), then what can you say about the numbers that can be written as the sum of 4 consecutive whole numbers, from the result of combining then factorising this expression? [1]
2. What do you notice about the middle 2 numbers? [1]
3. (7 marks)  
   Pick any 6 sets of **5** consecutive whole numbers and list their sums: [3}

|  |  |  |
| --- | --- | --- |
|  | Consecutive Numbers | Sum of consecutive numbers |
| Set 1 |  |  |
| Set 2 |  |  |
| Set 3 |  |  |
| Set 4 |  |  |
| Set 5 |  |  |
| Set 6 |  |  |

1. What do the sums have in common? [1]
2. What is the connection between the middle number and the sum? [1]
3. How can you calculate 41+42+43+44+45 without adding the numbers? [1]
4. If your 5 numbers are n, n+1, n+2, n+3 and n+4 then prove your discovery in a) [1]
5. (5 marks)  
   Starting with 3 + 4 , repeatedly **add a consecutive *integer* to both ends**. Repeat 5 times.

i.e. 3 + 4

2 + 3 + 4 + 5 (1st repeat)

Explain why the sums are always a multiple of 7.

**End of Investigation**